

Spin Nematic and Spin Density Wave Orders in Spatially Anisotropic Frustrated Magnets in Magnetic Fields

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(Dated: August 13, 2012)

We develop a theory for spin nematic ordering at finite temperatures in three-dimensional spatially anisotropic magnets consisting of weakly coupled frustrated spin- $\frac{1}{2}$ chains. This theory is applied for weakly coupled J_1 - J_2 chains with ferromagnetic nearest-neighbor J_1 and antiferromagnetic next-nearest-neighbor J_2 in magnetic fields. Combining the field theory technique with density-matrix renormalization group results, we complete the finite-temperature phase diagram in magnetic fields, which possesses spin bond-nematic and incommensurate spin-density-wave ordered phases. Effects of a four-spin coupling are also studied. The relevance of our result to quasi-one-dimensional edge-shared cuprate magnets such as LiCuVO_4 is discussed.

PACS numbers: 75.10.-b, 75.10.Jm, 75.10.Pq, 75.30.Fv, 75.40.Gb

Introduction.— Multipolar orders have long been studied in the context of heavy fermion systems. In recent years, their possible realization has been discussed much actively in quantum spin systems [1–7]. N -th order multipolar order parameters are here defined by the expectation values of symmetrized tensor products of N spin operators $\mathbf{S}_{\mathbf{r}_1}, \mathbf{S}_{\mathbf{r}_2}, \dots, \mathbf{S}_{\mathbf{r}_N}$. In the N -th multipolar ordered states, the N -th multipolar order parameter has a finite value, while all of the $M(< N)$ -th multipolar order parameters vanish. Among them, a quadrupolar state is called a spin nematic state, which has a finite quadrupolar order $\langle S_r^+ S_r^+ + \text{h.c.} \rangle \neq 0$ and zero dipolar moment $\langle \mathbf{S}_r \rangle = 0$. In usual magnets, however, a spin order with a finite local magnetization $\langle \mathbf{S}_r \rangle$ occurs at sufficiently low temperatures. Geometrical frustration is hence expected to be an important ingredient for the emergence of multipolar order [1].

In the spin- $\frac{1}{2}$ systems, multipolar operators cannot be defined in a single site because of the commutation relation of spin- $\frac{1}{2}$ operators. They reside on *bonds* between different sites [1, 3], which is a significant difference from the multipolar phases in heavy fermion or higher-spin systems [7]. From this property, it is generally hard to develop controllable, effective theories for multipolar phases in spin- $\frac{1}{2}$ magnets compared to those in other systems. Hence, most of the theoretical studies for multipolar phases in two- and three-dimensional (3D) spin- $\frac{1}{2}$ systems have been done by numerical computations for finite-size systems [1], and analytic theories for the multipolar phases, especially, in wide temperature and external-field ranges, have been less developed. Mean-field theories have been developed quite recently for the spin nematic ground state [8, 9].

In 1D spin- $\frac{1}{2}$ systems, on the other hand, various powerful theoretical/numerical techniques are applicable. Thanks to them, it has been shown recently that, in applied external magnetic field, three multipolar

Tomonaga-Luttinger (TL) liquid phases [3, 4] emerge in the spin- $\frac{1}{2}$ J_1 - J_2 chain with ferromagnetic (FM) nearest-neighbor coupling $J_1 < 0$ and antiferromagnetic (AF) next-nearest-neighbor one $J_2 > 0$, whose Hamiltonian is given as

$$\mathcal{H} = \sum_{n=1,2} \sum_j J_n \mathbf{S}_j \cdot \mathbf{S}_{j+n} - H \sum_j S_j^z. \quad (1)$$

Here \mathbf{S}_j is the spin- $\frac{1}{2}$ operator on site j , and H is the external field. Near the saturation, quadrupolar (nematic) $S_j^\pm S_{j+1}^\pm$, octupolar $S_j^\pm S_{j+1}^\pm S_{j+2}^\pm$, and hexadecapolar $S_j^\pm S_{j+1}^\pm S_{j+2}^\pm S_{j+3}^\pm$ operators exhibit quasi long-range order for the range $-2.7 \lesssim J_1/J_2 < 0$, $-3.5 \lesssim J_1/J_2 \lesssim -2.7$, and $-3.76 \lesssim J_1/J_2 \lesssim -3.5$, respectively, while the transverse spin correlator $\langle S_j^\pm S_0^\mp \rangle$ decays exponentially due to the formation of multiple-magnon bound states [3]. These multipolar TL liquid phases expand down to a low-field regime, where the dominant correlation turns to an incommensurate longitudinal spin density wave (SDW) type.

The J_1 - J_2 spin chain is expected to be an effective model for a series of quasi-1D edge-shared cuprate magnets such as LiCuVO_4 [10–15], $\text{Rb}_2\text{Cu}_2\text{Mo}_3\text{O}_{12}$ [16], $\text{PbCuSO}_4(\text{OH})_2$ [17, 18], LiCuSbO_4 [19] and LiCu_2O_2 [20]. The experimentally estimated coupling ratio J_1/J_2 for LiCuVO_4 [10] is well inside of the spin nematic TL-liquid phase in the J_1 - J_2 chain. These theoretical and experimental results have motivated further searches of the spin nematic quasi and true long-range ordered phases in the high-field regime of LiCuVO_4 [11, 15]. In addition, recent experiments in the intermediate field regime found incommensurate SDW oscillations [12–14] whose wave vector nicely agrees with the theoretical prediction for the TL liquids of two-magnon bound states [2, 3, 5]. It is still obscure how 3D spin nematic and SDW ordered phases are induced with lowering temperature in quasi-1D magnets with

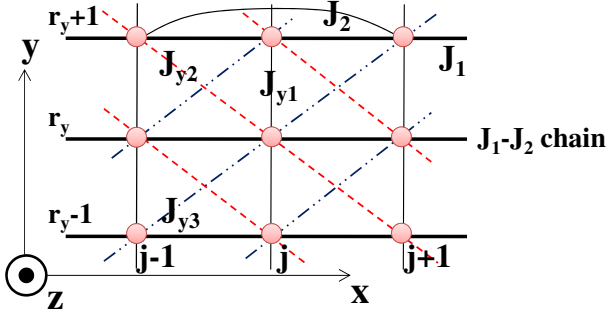


FIG. 1: (color online) Spatially anisotropic spin model consisting of weakly coupled spin- $\frac{1}{2}$ J_1 - J_2 chains. We introduce inter-chain couplings J_{y1} , J_{y2} , J_{y3} in the x - y plane. Similarly, J_{z1} , J_{z2} , J_{z3} are present in the x - z plane.

weak interchain couplings, and how both of them are described in a unified way.

In this paper, we develop a general theory for spin nematic and incommensurate SDW orders in spatially anisotropic 3D magnets consisting of weakly coupled J_1 - J_2 spin chains in a *wide magnetic-field range*. Making use of field theoretical and numerical results for the J_1 - J_2 spin chain, we obtain *finite temperature* phase diagrams, which contain both spin nematic and incommensurate SDW phases at sufficiently low temperatures. From them, we reveal some characteristic features in the ordering of weakly coupled J_1 - J_2 chains. We also discuss the relevance of our results to real compounds such as LiCuVO_4 .

Model.— Now, we start with the definition of our model of spatially anisotropic magnets shown in Fig. 1. The corresponding Hamiltonian is expressed as

$$\mathcal{H}_{3D} = \sum_{\mathbf{r}} \mathcal{H}_{\mathbf{r}} + \mathcal{H}_{\text{int}}, \quad (2)$$

where $\mathbf{r} = (r_y, r_z)$ denotes the site index of the square lattice in the y - z plane, $\mathcal{H}_{\mathbf{r}}$ denotes the Hamiltonian (1) for the \mathbf{r} -th J_1 - J_2 chain along the x axis in magnetic field H , and \mathcal{H}_{int} is the inter-chain interaction. In \mathcal{H}_{int} , we introduce weak inter-chain Heisenberg-type exchange interactions with coupling constants J_{y_i} and J_{z_i} ($i = 1, 2, 3$) defined in the x - y and x - z planes, respectively [21].

Spin- $\frac{1}{2}$ J_1 - J_2 chain.— Under the condition $|J_{y_i, z_i}| \ll |J_{1,2}|$, decoupled J_1 - J_2 spin chains $\mathcal{H}_{\mathbf{r}}$ may be chosen as the starting point for analyzing the 3D model \mathcal{H}_{3D} . Static and dynamic properties of the multipolar TL liquids in the J_1 - J_2 chain $\mathcal{H}_{\mathbf{r}}$ have been well studied [3–6]. The low-energy effective Hamiltonian for the spin nematic TL liquid phase ($-2.7 \lesssim J_1/J_2 < 0$) is given by

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{\mathbf{r}} = & \int dx \sum_{\nu=\pm} \frac{v_{\nu}}{2} [K_{\nu}(\partial_x \theta_{\nu}^{\mathbf{r}})^2 + K_{\nu}^{-1}(\partial_x \phi_{\nu}^{\mathbf{r}})^2] \\ & + G_{-} \sin(\pi M) \sin(\sqrt{4\pi} \phi_{-}^{\mathbf{r}} + \pi M), \end{aligned} \quad (3)$$

where $x = a_0 j$ (the length a_0 of J_1 bond is set equal to unity), $(\phi_{\pm}^{\mathbf{r}}(x), \theta_{\pm}^{\mathbf{r}}(x))$ is the canonical pair of scalar boson fields, and v_{\pm} and K_{\pm} are, respectively, the excitation velocity and TL-liquid parameter of the $(\phi_{\pm}, \theta_{\pm})$ sector. The term with the coupling G_{-} makes ϕ_{-} pinned, inducing an excitation gap in the (ϕ_{-}, θ_{-}) sector. Physically, the gap corresponds to the magnon binding energy E_b . On the other hand, the (ϕ_{+}, θ_{+}) sector is described by a massless TL liquid. Vertex operators are renormalized as $\langle e^{i\alpha\sqrt{\pi}\phi_{+}(x)} e^{-i\alpha\sqrt{\pi}\phi_{+}(0)} \rangle_{+} = |2/x|^{\alpha^2 K_{+}/2}$ for $|x| \gg 1$, in which $\langle \cdots \rangle_{\pm}$ is the expectation value of the $(\phi_{\pm}, \theta_{\pm})$ sector.

Spin operators $S_{j,\mathbf{r}}$ are also bosonized as

$$\begin{aligned} S_{j,\mathbf{r}}^z \approx & M + \partial_x(\phi_{+}^{\mathbf{r}} + (-1)^j \phi_{-}^{\mathbf{r}})/\sqrt{\pi} \\ & + (-1)^q A_1 \cos[\sqrt{\pi}(\phi_{+}^{\mathbf{r}} + (-1)^j \phi_{-}^{\mathbf{r}}) + 2\pi M q] + \cdots, \end{aligned} \quad (4a)$$

$$\begin{aligned} S_{j,\mathbf{r}}^{\pm} \approx & e^{i\sqrt{\pi}(\theta_{+}^{\mathbf{r}} + (-1)^j \theta_{-}^{\mathbf{r}})} \{ (-1)^q B_0 \\ & + B_1 \cos[\sqrt{\pi}(\phi_{+}^{\mathbf{r}} + (-1)^j \phi_{-}^{\mathbf{r}}) + 2\pi M q] + \cdots \}, \end{aligned} \quad (4b)$$

where $M = \langle S_{j,\mathbf{r}}^z \rangle$, $q = \frac{j}{2}$ ($\frac{j-1}{2}$) for $j = \text{even}$ (odd), and both A_n and B_n are nonuniversal constants. Utilizing Eqs. (3) and (4), we can evaluate spin and nematic correlation functions at zero temperature ($T = 0$) as follows [3, 5, 6]:

$$\langle S_j^+ S_0^- \rangle \approx B_0^2 \cos(\pi j/2) (2/|j|)^{1/(2K_{+})} g_{-}(x) + \cdots, \quad (5a)$$

$$\begin{aligned} \langle S_j^z S_0^z \rangle \approx & M^2 + (A_1^2/2) |\langle e^{i\sqrt{\pi}\phi_{-}} \rangle_-|^2 \\ & \times \cos[\pi j(M - 1/2)] (2/|j|)^{K_{+}/2} + \cdots, \end{aligned} \quad (5b)$$

$$\langle S_j^+ S_{j+1}^+ S_0^- S_1^- \rangle \approx (-1)^j C_0 |j|^{-2/K_{+}} + \cdots, \quad (5c)$$

where $g_{-}(x) = \langle e^{\pm i\sqrt{\pi}\theta_{-}(x)} e^{\mp i\sqrt{\pi}\theta_{-}(0)} \rangle_{-}$, C_0 is a constant and we have omitted the index \mathbf{r} . The function $g_{-}(x)$ decays in an exponential fashion like $\sim x^{-1/2} e^{-x/\xi_{-}}$. The TL-liquid parameter K_{+} , which is less than 2 in the low magnetization regime, monotonically increases as a function of M [3] and $K_{+} \rightarrow 4$ at the saturation. Thus, the spin nematic correlation is stronger (weaker) than the incommensurate SDW correlation in high-field (low-field) regime with $K_{+} > 2$ ($K_{+} < 2$).

The correlation length ξ_{-} is related to v_{-} via $v_{-} = \xi_{-} E_b$ under the assumption that the low-energy theory for the (ϕ_{-}, θ_{-}) sector has Lorentz invariance. The velocity v_{+} has the relation $v_{+} = 2K_{+}/(\pi\chi)$, where $\chi = \partial M/\partial H$ is the uniform susceptibility. The values of K_{+} , ξ_{-} , E_b , and χ are all determined with reasonable accuracy by using density-matrix renormalization group (DMRG) method [3, 22]. Thus, v_{\pm} can be quantitatively evaluated as depicted in Fig. 2. The figure shows that v_{-} is always larger than v_{+} , and it is consistent with the perturbative formula $v_{\pm} \approx v(1 \pm K J_1/(\pi v) + \cdots)$ in the weak $|J_1|/J_2$ regime, in which v and K are respectively the spinon velocity and the TL-liquid parameter for the

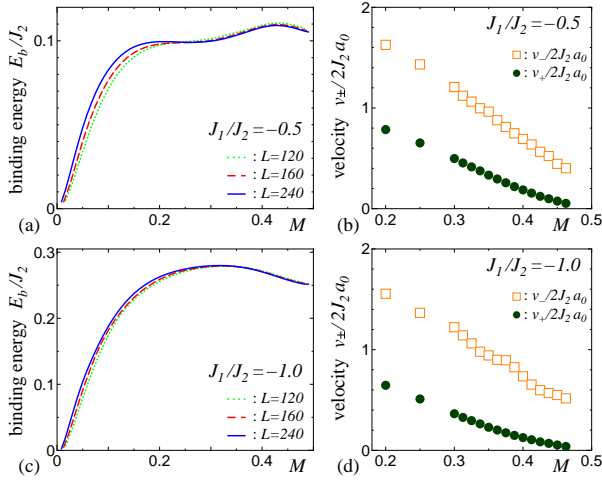


FIG. 2: (color online) Magnon binding energy E_b (a)(c) and excitation velocities v_{\pm} (b)(d) as a function of M in the spin-nematic TL liquid phase in the spin- $\frac{1}{2}$ J_1 - J_2 chain at $T = 0$.

single AF- J_2 chain. We also note that v_+ approaches to zero in the vicinity of the saturation $M \rightarrow \frac{1}{2}$.

Analysis of the 3D model.— Let us now analyze the 3D model (2) starting with the effective theory of the J_1 - J_2 chain. We first bosonize all of the inter-chain couplings in \mathcal{H}_{int} through Eq. (4). To obtain the low-energy effective theory for Eq. (2), we trace out the massive (ϕ_-^r, θ_-^r) sectors in the Euclidean action $\mathcal{S}_{\text{tot}} = \mathcal{S}_0 + \mathcal{S}_{\text{int}}$ via the cumulant expansion $\mathcal{S}_{\text{eff}}^{3D} = \mathcal{S}_0 + \langle \mathcal{S}_{\text{int}} \rangle_- - \frac{1}{2} (\langle \mathcal{S}_{\text{int}}^2 \rangle_- - \langle \mathcal{S}_{\text{int}} \rangle_-^2) + \dots$, where \mathcal{S}_0 and \mathcal{S}_{int} are respectively the action for the TL-liquid part of the (ϕ_+^r, θ_+^r) sectors and that for the inter-chain couplings. This expansion corresponds to the series expansion in $J_{y_i, z_i}/v_-$. The resultant effective Hamiltonian is expressed as $\mathcal{H}_{\text{eff}}^{3D} = \mathcal{H}_0 + \mathcal{H}_{\text{SDW}} + \mathcal{H}_{\text{Ne}} + \dots$. Here, $\mathcal{H}_0 = \sum_{\mathbf{r}} \int dx \frac{v_+}{2} [K_+ (\partial_x \theta_+^r)^2 + K_+^{-1} (\partial_x \phi_+^r)^2]$ is the TL-liquid part, and \mathcal{H}_{SDW} and \mathcal{H}_{Ne} are, respectively, obtained from the first- and second-order cumulants as follows:

$$\mathcal{H}_{\text{SDW}} = G_{\text{SDW}} \int \frac{dx}{2} \sum_{\mathbf{r}} \sum_{\substack{\alpha=y,z \\ (\mathbf{r}'=\mathbf{r}+\mathbf{e}_{\alpha})}} \left[J_{\alpha 1} \cos(\sqrt{\pi}(\phi_+^{\mathbf{r}} - \phi_+^{\mathbf{r}'})) - J_{\alpha 2} \sin(\sqrt{\pi}(\phi_+^{\mathbf{r}} - \phi_+^{\mathbf{r}'} - \pi M)) + J_{\alpha 3} \sin(\sqrt{\pi}(\phi_+^{\mathbf{r}} - \phi_+^{\mathbf{r}'} + \pi M)) \right], \quad (6a)$$

$$\mathcal{H}_{\text{Ne}} = G_{\text{Ne}} \int \frac{dx}{2} \sum_{\mathbf{r}} \sum_{\substack{\alpha=y,z \\ (\mathbf{r}'=\mathbf{r}+\mathbf{e}_{\alpha})}} [J_{\alpha 1}^2 - (J_{\alpha 2} - J_{\alpha 3})^2] \times \cos(\sqrt{4\pi}(\theta_+^{\mathbf{r}} - \theta_+^{\mathbf{r}'})) \quad (6b)$$

with coupling constants $G_{\text{SDW}} = A_1^2 |\langle e^{i\sqrt{\pi}\phi_-} \rangle_-|^2$ [23] and $G_{\text{Ne}} = -\frac{B_0^4}{4v_-} \int dx v_- d\tau g_-(x, \tau)^2$ (τ is imaginary time). The summations run over all nearest neighbor

pairs of chains, where $\mathbf{r}' = \mathbf{r} + \mathbf{e}_{\alpha}$ ($\alpha = y, z$), \mathbf{e}_{α} denotes the unit vector along the α -axis, and we have assumed that the field ϕ_+ smoothly varies in x . The first-order term \mathcal{H}_{SDW} contains an inter-chain interaction between the operators $e^{\pm i\sqrt{\pi}\phi_+^r}$, which essentially induces a 3D spin longitudinal order. Similarly, the term \mathcal{H}_{Ne} contains an inter-chain interaction between the spin nematic operators $S_{j,\mathbf{r}}^{\pm} S_{j+1,\mathbf{r}}^{\pm} \sim (-1)^j e^{\pm i\sqrt{4\pi}\theta_+^r}$, which enhances 3D spin nematic correlation. We should notice that the effective theory $\mathcal{H}_{\text{eff}}^{3D}$ is reliable under the condition that temperature T is sufficiently smaller than the binding energy E_b and the velocities v_{\pm} .

Both the couplings $G_{\text{SDW}, \text{Ne}}$ can be numerically evaluated by using the correlation functions estimated with DMRG method [3, 22]: G_{SDW} corresponds to the amplitude of the leading term of the longitudinal correlator $\langle S_j^z S_0^z \rangle$ given in Eq. (5) and G_{Ne} can be evaluated as $G_{\text{Ne}} \approx \pi v_-^{-1} \sum_{j=1}^L (j/2)^{1/K_+} j \langle S_j^+ S_0^- \rangle^2$. We have checked that the finite size-correction to the sum is small enough when the cut off L is larger than ξ_- . We emphasize that *there is no free parameter in the field-theoretical Hamiltonian $\mathcal{H}_{\text{eff}}^{3D}$.*

To obtain the finite-temperature phase diagram, we apply the inter-chain mean-field (ICMF) approximation [24, 25] to the effective Hamiltonian $\mathcal{H}_{\text{eff}}^{3D}$. To this end, we introduce the "effective" SDW operator $\mathcal{O}_{\text{SDW}} = e^{i\pi(\frac{1}{2}-M)j} e^{i\sqrt{\pi}\phi_+^r}$ and the spin nematic operator $\mathcal{O}_{\text{Ne}} = (-1)^j e^{i\sqrt{4\pi}\theta_+^r}$. Within the ICMF approach, the finite-temperature dynamical susceptibilities of \mathcal{O}_A ($A = \text{SDW}$ or Ne) above 3D ordering temperatures are calculated as

$$\chi_A(k_x, \mathbf{k}, \omega) = \frac{\chi_A^{1D}(k_x, \omega)}{1 + J_{\text{eff}}^A(\mathbf{k}) \chi_A^{1D}(k_x, \omega)}, \quad (7)$$

where $\mathbf{k} = (k_y, k_z)$ is the wave vector in the y - z plane, ω is the frequency, and the effective coupling constants J_{eff}^A are given by

$$J_{\text{eff}}^{\text{SDW}}(\mathbf{k}) = G_{\text{SDW}} \sum_{\alpha=y,z} [J_{\alpha 1} \cos k_{\alpha} - J_{\alpha 2} \sin(k_{\alpha} - \pi M) + J_{\alpha 3} \sin(k_{\alpha} + \pi M)], \quad (8a)$$

$$J_{\text{eff}}^{\text{Ne}}(\mathbf{k}) = G_{\text{Ne}} \sum_{\alpha=y,z} [J_{\alpha 1}^2 - (J_{\alpha 2} - J_{\alpha 3})^2] \cos k_{\alpha}. \quad (8b)$$

The 1D susceptibilities $\chi_A^{1D}(k_x, \omega) = \frac{1}{2} \sum_j e^{-ik_x j} \int_0^{\beta} d\tau e^{i\omega_n \tau} \langle \mathcal{O}_A(j, \tau) \mathcal{O}_A^{\dagger}(0, 0) \rangle |_{i\omega_n \rightarrow \omega + i\epsilon}$ are analytically computed by using field theory technique ($\beta = 1/T$ and $\epsilon \rightarrow +0$) [26]. Those for SDW and spin nematic operators respectively take the maximum value at $k_x^{\text{max}} = (\frac{1}{2} - M)\pi$ and π ; $\chi_{\text{SDW}}^{1D}(k_x^{\text{max}}, 0) = \frac{2}{v_+} (\frac{4\pi}{\beta v_+})^{K_+/2-2} \sin(\frac{\pi K_+}{4}) B(\frac{K_+}{8}, 1 - \frac{K_+}{4})^2$ and $\chi_{\text{Ne}}^{1D}(\pi, 0) = \frac{2}{v_+} (\frac{4\pi}{\beta v_+})^{2/K_+-2} \sin(\frac{\pi}{K_+}) B(\frac{1}{2K_+}, 1 - \frac{1}{K_+})^2$, where $B(x, y)$ is beta function.

The transition temperature of each order is obtained from the divergent point of its own susceptibility at $\omega \rightarrow$

0, which is given by

$$1 + \text{Min}_{\mathbf{k}}[J_{\text{eff}}^A(\mathbf{k})]\chi_A^{1D}(k_x^{\text{max}}, 0) = 0. \quad (9)$$

The 3D ordered phase with the highest transition temperature T_c is realized below T_c . From this ICMF scheme, we can determine the phase diagram for \mathcal{H}_{3D} with arbitrary combination of weak inter-chain couplings J_{y_i, z_i} . We should note that, when J_{eff}^A approaches to zero, the present framework becomes less reliable and we need to consider sub-leading terms in $\mathcal{H}_{\text{eff}}^{3D}$.

From Eqs. (8) and (9), we find that the ordering wave numbers $k_{y,z}$ tend to be a commensurate value $k_{y,z} = 0$ or π (see also the comment in Ref. 23). Thus the SDW ordered phase has the wave vector $k_x = (\frac{1}{2} - M)\pi$ and $k_{y,z} = 0$ or π . This agrees with the experimental result in the intermediate-field phase of LiCuVO_4 [12, 13]. For the spin nematic ordered phase, we find the commensurate ordering vector $(k_x, k_{y(z)}) = (\pi, 0)$ for $|J_{y_1(z_1)}| > |J_{y_2(z_2)} - J_{y_3(z_3)}|$ and $(k_x, k_{y(z)}) = (\pi, \pi)$ for $|J_{y_1(z_1)}| < |J_{y_2(z_2)} - J_{y_3(z_3)}|$.

We show some typical examples of obtained phase diagrams in Fig. 3. When interchain couplings are not frustrated such as the $J_{y_1}(J_{z_1})$ dominant case of Fig. 3(a) and (b), the SDW ordered phase is largely enhanced and the nematic ordered phase is reduced to a higher-field regime compared to the crossover line ($K_+ = 2$) in the J_1 - J_2 chain. This is because the effective couplings $J_{\text{eff}}^{\text{SDW}}$ and $J_{\text{eff}}^{\text{Ne}}$ are respectively generated from the first- and second-order cumulants, and therefore $J_{\text{eff}}^{\text{SDW}}$ is generally larger than $J_{\text{eff}}^{\text{Ne}}$ in non-frustrated systems in a sufficiently weak interchain coupling regime. When both the couplings J_{y_2} and J_{y_3} are dominant, we find the similar tendency. The systems with dominant J_{y_2} and J_{y_3} resemble the experimental proposal for LiCuVO_4 [10], where a new phase expected to be a 3D nematic phase has been observed only near the saturation [11]. From the calculations for the cases of $|J_1|/J_2 = 0.5, 1.0$, and 2.0 , we find that the nematic phase region in the M - T phase diagram generally becomes smaller with increase in $|J_1|/J_2$ since the value $g_-(x)$ in G_{Ne} decreases. The shrinkage of the nematic phase was also discussed in the ground state of multi-leg J_1 - J_2 ladders [27]. When there is a certain frustration in interchain couplings, however, the nematic phase region can expand, as shown in Fig. 3(c). When the signs of J_{y_1} and $J_{y_2}(J_{y_3})$ are opposite, the contribution to the coupling $J_{\text{eff}}^{\text{SDW}}$ from the first-order perturbation becomes very weak, which expands the 3D nematic ordered phase down to a relatively lower-field regime.

Effects of four-spin term.— Finally, we study effects of an inter-chain four-spin interaction on the phase diagram. The four-spin term we consider is

$$\mathcal{H}_4 = -J_4 \sum_{j, \langle \mathbf{r}, \mathbf{r}' \rangle} S_{j, \mathbf{r}}^+ S_{j+1, \mathbf{r}}^+ S_{j, \mathbf{r}'}^- S_{j+1, \mathbf{r}'}^- + \text{h.c.} \quad (10)$$

This interaction can be regarded as a part of the spin-phonon coupling $\mathcal{H}_{\text{sp}} = -J_{\text{sp}} \sum_{j, \langle \mathbf{r}, \mathbf{r}' \rangle} (\mathbf{S}_{j, \mathbf{r}} \cdot$

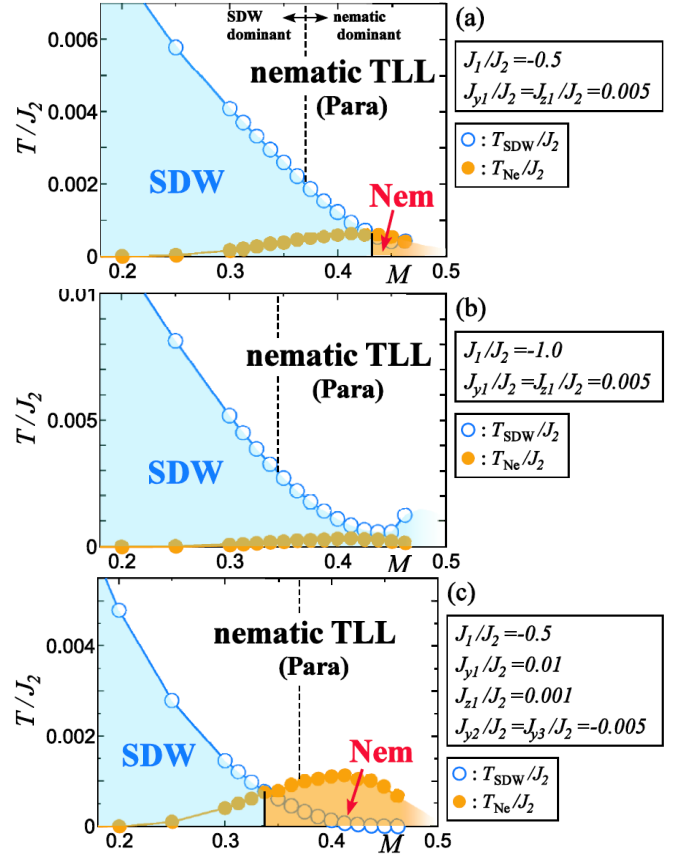


FIG. 3: (color online) Phase diagrams of the 3D magnets (2) of weakly coupled J_1 - J_2 chains in the M - T plane, which are derived by the ICMF approach. The temperatures $T_{\text{SDW(Ne)}}$ denote the 3D SDW (nematic) transition points. The vertical dashed lines denote the crossover lines in the 1D J_1 - J_2 chain between nematic dominant and SDW dominant TL liquids.

$\mathbf{S}_{j, \mathbf{r}'})(\mathbf{S}_{j+1, \mathbf{r}} \cdot \mathbf{S}_{j+1, \mathbf{r}'}).$ One can easily expect that Eq. (10) enhances the spin nematic ordering. Applying the field theory strategy to the system $\mathcal{H}_{3D} + \mathcal{H}_4$, we find that $J_{\text{eff}}^{\text{Ne}}$ is replaced with $J_{\text{eff}}^{\text{Ne}} - 4J_4 C_0 (\cos k_y + \cos k_z)$. We thus obtain the phase diagram for $\mathcal{H}_{3D} + \mathcal{H}_4$, as shown in Fig. 4. Comparing Fig. 3(a) [(b)] and Fig. 4(a) [(b)], we see that an inter-chain four-spin interaction definitely enhances the 3D nematic phase even if its coupling constant J_4 is small. Since J_4 is usually negative, it favors ferro-type nematic ordering along the y and z axes, i.e., $k_{y,z} = 0$.

Conclusion.— We have constructed finite-temperature phase diagrams for 3D spatially anisotropic magnets, which consist of weakly coupled spin- $\frac{1}{2}$ J_1 - J_2 chains, in applied magnetic field. Incommensurate SDW and spin nematic ordered phases appear at sufficiently low temperatures, triggered by the critical TL liquid properties in the J_1 - J_2 spin chains. We reveal several natures of orderings of coupled J_1 - J_2 chains: The 3D nematic ordered phase is generally smaller than 1D nematic domi-

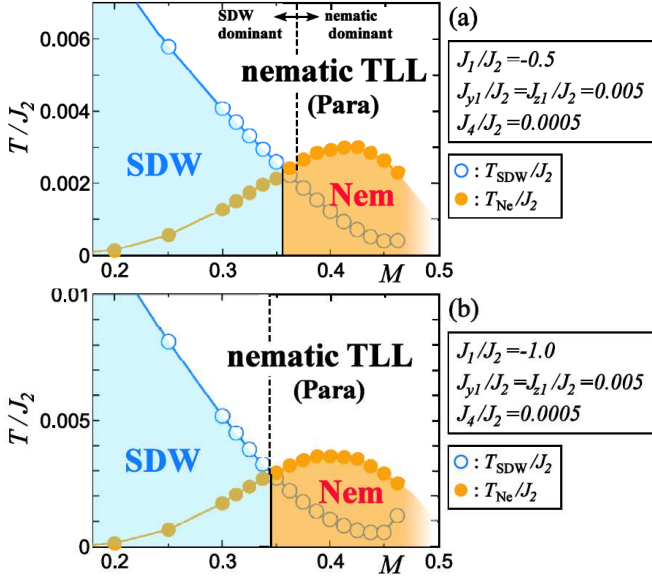


FIG. 4: (color online) Phase diagrams of the weakly coupled J_1 - J_2 spin chains (2) with a weak inter-chain four-spin interaction \mathcal{H}_4 .

nant region, while it can be larger if we somewhat tune the inter-chain couplings. The ordering wave numbers $k_{y,z}$ tend to be 0 or π , and a small four-spin interaction \mathcal{H}_4 efficiently helps the 3D nematic ordering.

We thank Akira Furusaki for fruitful discussions at the early stage of this study. This work was supported by KAKENHI No. 21740295, No. 22014016, and No. 23540397 from MEXT, Japan.

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